10.2

$x = f(t)$
$y = g(t)$

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{g'(t)}{f'(t)}$$

eg. $x = 4t - t^3 = t(4-t^2) \longrightarrow$ y-INT $\rightarrow x=0 \rightarrow t = 0, \pm 2$
$y = t^2 - 1$ $\qquad\qquad\qquad\qquad y = -1, 3$

$$\frac{dy}{dx} = \frac{2t}{4-3t^2}$$

FIND ALL POINTS WHERE T.L. IS VERTICAL

SLOPE UNDEFINED

$4 - 3t^2 = 0$

IE. $\frac{dx}{dt} = 0 \longrightarrow t = \pm\frac{2}{\sqrt{3}}$

$t = \frac{2}{\sqrt{3}}$
$x = \frac{2}{\sqrt{3}}(4 - \frac{4}{3}) = \frac{16}{3\sqrt{3}}$
$y = \frac{4}{3} - 1 = \frac{1}{3}$
$\left(\frac{16}{3\sqrt{3}}, \frac{1}{3}\right)$

$t = -\frac{2}{\sqrt{3}}$
$x = \frac{-16}{3\sqrt{3}}$
$y = \frac{1}{3}$
$\left(\frac{-16}{3\sqrt{3}}, \frac{1}{3}\right)$

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dt}\left(\frac{dy}{dx}\right)\frac{dt}{dx}$$

$$\frac{d^2y}{dx^2} = \frac{\frac{d}{dt}\left(\frac{dy}{dx}\right)}{\frac{dx}{dt}} \neq \frac{g''(t)}{f''(t)}$$

ALSO $\neq \left(\frac{dy}{dx}\right)'$

FIND $\frac{d^2y}{dx^2}$ FOR $\begin{array}{l} x = 4t - t^3 \\ y = t^2 - 1 \end{array}$

$$\frac{\frac{d}{dt}\left(\frac{dy}{dx}\right)}{\frac{dx}{dt}} = \frac{\frac{d}{dt}\,\frac{2t}{4-3t^2}}{4-3t^2} = \frac{\frac{2(4-3t^2)-2t(-6t)}{(4-3t^2)^2}}{4-3t^2}$$

$$= \frac{8+6t^2}{(4-3t^2)^3}$$

FIND CONCAVITY @ y-INTS OF

$$\left.\frac{d^2y}{dx^2}\right|_{t=0} = \frac{8+0}{(4-0)^3} = \frac{8}{64} = \frac{1}{8} \quad \text{CONCAVE UP}$$

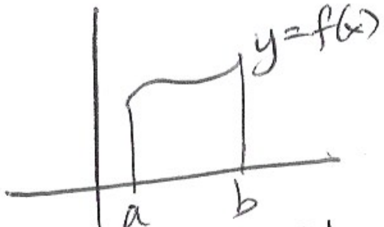
$$\left.\frac{d^2y}{dx^2}\right|_{t=\pm 2} = \frac{8+24}{(4-12)^3} = \frac{\cancel{32}\,4}{-\cancel{8}\cdot 8\cdot 8\;2} = -\frac{1}{16} \quad \text{CONCAVE DOWN}$$

BTW $\frac{d^2y}{dx^2}$ CAN BE WRITTEN IN TERMS OF f, g, f', g'

BUT NOT EASILY

AREA UNDER CURVE

1B



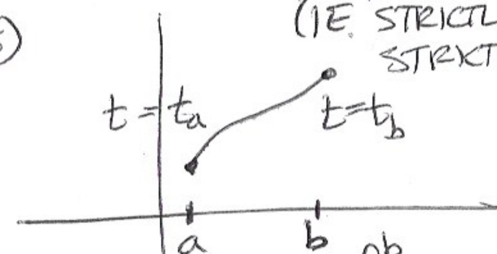
$$A = \int_a^b f(x)\,dx$$

$$\downarrow$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} \underbrace{f(x_i)}_{y_i} \Delta x$$

$x = f(t)$
$y = g(t)$

CURVE ABOVE X-AXIS
+ DOES NOT "DOUBLE BACK"
(IE STRICTLY TO RIGHT OR
STRICTLY TO LEFT)



$$A = \int_a^b y\,dx \quad (a<b)$$

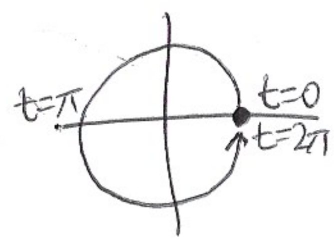
$$= \int_{t_a}^{t_b} g(t)\, f'(t)\,dt$$

$\cos 2t = 1 - 2\sin^2 t$

$\sin^2 t = \frac{1}{2}(1-\cos 2t)$

FIND THE AREA OF A CIRCLE OF RADIUS a CENTERED @ ORIGIN
USING PARAMETRIC EQUATIONS

$x = a\cos t$
$y = a\sin t$
$t \in [0, 2\pi]$

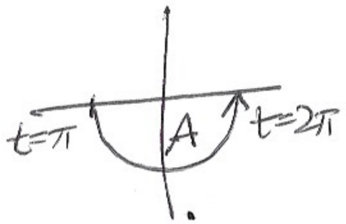


$$\int_\pi^0 a\sin t \cdot -a\sin t\,dt = a^2\int_0^\pi \sin^2 t\,dt$$

$$= a^2\int_0^\pi \tfrac{1}{2}(1-\cos 2t)\,dt$$

$$= \tfrac{1}{2}a^2\left(t - \tfrac{1}{2}\sin 2t\right)\Big|_0^\pi$$

$$= \tfrac{1}{2}a^2(\pi - 0) = \tfrac{1}{2}\pi a^2$$

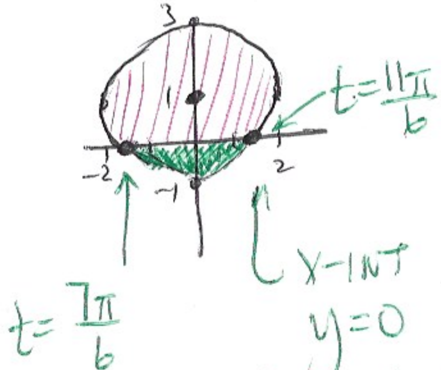


$$-A = \int_{\pi}^{2\pi} a\sin t \cdot -a\sin t\,dt$$

$$A = a^2\int_{\pi}^{2\pi} \sin^2 t\,dt = \frac{1}{2}a^2\left(t - \frac{1}{2}\sin 2t\right)\Big|_{\pi}^{2\pi} = \frac{1}{2}a^2(2\pi - \pi) = \frac{1}{2}\pi a$$

TOTAL $\frac{1}{2}\pi a^2 + \frac{1}{2}\pi a^2 = \pi a^2$

FIND AREA OF PORTION OF CIRCLE OF RADIUS 2 WITH CENTER @ (0,1) THAT IS ABOVE X-AXIS



$x = 2\cos t$
$y = 1 + 2\sin t$

X-INT
$y=0$
$1+2\sin t=0$
$\sin t=-\frac{1}{2}$
$t=\frac{7\pi}{6}, \frac{11\pi}{6}$

$$A = -\int_{\frac{7\pi}{6}}^{\frac{11\pi}{6}} (1+2\sin t)(-2\sin t)\,dt$$

$$= 2\int_{\frac{7\pi}{6}}^{\frac{11\pi}{6}} (\sin t + 2\sin^2 t)\,dt$$

$$= 2\left(-\cos t + 1 - \cos 2t\right)\Big|_{\frac{7\pi}{6}}^{\frac{11\pi}{6}}$$

IGNORE IN

$$= 2\left(-\frac{\sqrt{3}}{2} - \frac{1}{2} - \left(\frac{\sqrt{3}}{2} - \frac{1}{2}\right)\right)$$

$$= -\sqrt{3} - 1 - \sqrt{3} + 1 = -2\sqrt{3}$$